Homework assignment #8
(due Friday, November 10)

Problem 1. Solve
$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + e^{-2t}\sin 5x$$
subject to $u(0,t) = 1$, $u(\pi,t) = 0$, and $u(x,0) = 0$.

Problem 2. Solve
$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$
subject to $u(0,t) = 0$, $u(L,t) = t$, and $u(x,0) = 0$.

Problem 3. Solve the following example of Poisson's equation:
$$\nabla^2 u = e^{2y}\sin x \qquad (0 < x < \pi,\ 0 < y < L)$$
subject to the following boundary conditions:
$$u(0,y) = u(\pi,y) = 0, \qquad u(x,0) = 0, \qquad u(x,L) = f(x).$$

Problem 4. Consider
$$f(x) = \begin{cases} 0, & x < x_0, \\ 1/\Delta, & x_0 < x < x_0 + \Delta, \\ 0, & x > x_0 + \Delta. \end{cases}$$
Assume that $-L < x_0 < x_0 + \Delta < L$. Determine the complex Fourier coefficients c_n of the function f on the interval $[-L, L]$.

Problem 5. If $F(\omega)$ is the Fourier transform of $f(x)$, show that the inverse Fourier transform of $e^{i\omega\beta}F(\omega)$ is $f(x-\beta)$. This result is known as the *shift theorem* for Fourier transforms.