## Sample problems for Exam 1

Any problem may be altered, removed or replaced by a different one!

**Problem 1.** Consider an operation \* defined on the set  $\mathbb{Z}$  of integers by a \* b = a + b - 2. Does this operation provide the integers with a group structure?

**Problem 2.** Suppose (S, \*) is a semigroup satisfying the following two conditions: (i) there exists  $e \in S$  such that e \* g = g for all  $g \in S$  (existence of a left identity element), and (ii) for any  $g \in S$  there exists  $g' \in S$  such that g' \* g = e (existence of a left inverse). Prove that (S, \*) is a group.

**Problem 3.** Prove that the group  $(\mathbb{Q} \setminus \{0\}, \cdot)$  is not cyclic.

**Problem 4.** Let G be a group of order 125. Show that G contains an element of order 5.

**Problem 5.** Find the order and the sign of the permutation  $\sigma = (1\ 2)(3\ 4\ 5\ 6)(1\ 2\ 3\ 4)(5\ 6)$ .

**Problem 6.** Suppose  $\pi, \sigma \in S_5$  are permutations of order 3. What are possible values for the order of the permutation  $\pi\sigma$ ?

**Problem 7.** Find all subgroups of the alternating group  $A_4$ .

**Problem 8.** Determine which of the following groups of order 12 are isomorphic and which are not:  $\mathbb{Z}_{12}$ ,  $\mathbb{Z}_3 \times \mathbb{Z}_4$ ,  $\mathbb{Z}_2 \times \mathbb{Z}_6$ ,  $S_3 \times \mathbb{Z}_2$ ,  $D_3 \times \mathbb{Z}_2$  and  $A_4$ .

**Problem 9.** Find an example of an abelian group G and its subgroups  $H_1$  and  $H_2$  such that the subgroups  $H_1$  and  $H_2$  are isomorphic while the factor groups  $G/H_1$  and  $G/H_2$  are not.

**Problem 10.** Complete the following Cayley table of a group of order 9:

*	A	B	C	D	E	F	G	H	Ι
A	Ι								F
В		F						G	
C			Н				E		
D				G		A			
E					E				
F				A		В			
G			E				A		
H		G						D	
Ι	F								C