

Sample problems for Exam 3

Any problem may be altered, removed or replaced by a different one!

Problem 1. Suppose $\pi, \sigma \in S(5)$ are permutations of order 3. What are possible values for the order of the permutation $\pi\sigma$.

Problem 2. Suppose H and K are subgroups of a group G . Is the union $H \cup K$ necessarily a subgroup of G ? Is the intersection $H \cap K$ necessarily a subgroup of G ?

Problem 3. Prove that the group $(\mathbb{Q} \setminus \{0\}, \times)$ is not cyclic.

Problem 4. Suppose G is a group of order 125. Show that G contains an element of order 5.

Problem 5. The group (G_{15}, \times) has subgroups of what orders?

Problem 6. For any two of the following groups of order 6, determine whether they are isomorphic or not: \mathbb{Z}_6 , $\mathbb{Z}_3 \times \mathbb{Z}_2$, $S(3)$, and $D(3)$.

Problem 7. Let $f : \mathbf{B}^3 \rightarrow \mathbf{B}^7$ be the coding function that sends each three-character word abc in the alphabet $\mathbf{B} = \{0, 1\}$ to the codeword $abcabcy$, where y is the inverted parity bit of the word abc (i.e., $y = 0$ if abc contains an odd number of 1's and $y = 1$ otherwise). How many errors will this code detect? correct? Is this code linear?

Problem 8. Let $f : \mathbf{B}^3 \rightarrow \mathbf{B}^6$ be a linear coding function defined by the generator matrix

$$\begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix}.$$

Suppose that a message encoded by this function is received with errors as

$$101101 \quad 010101 \quad 011111.$$

Correct errors and decode the received message.

Problem 9. Find a greatest common divisor of polynomials $p(x) = x^4 - 2x^3 + 5x^2 - 4x + 4$ and $q(x) = 2x^3 - 3x^2 + 5x - 2$ over \mathbb{R} .

Problem 10. Factorise a polynomial $p(x) = x^3 - 3x^2 + 3x - 2$ into irreducible factors over the field \mathbb{Z}_7 .