MATH 251 Exam 1 Review

WARNING: To success on the test it is not sufficent to work these problems only. It is advised that you also

- read over the lecture notes;
- work the quizzes
- work the suggested and webassign homework problems.

1. Given $z = \ln(x^2 + y^2 - 9)$.
   (a) Find the first partial derivatives of the given function at the point $(3, 1)$.
   (b) Find the gradient of the given function at $(3, 1)$.
   (c) Find the directional derivative of the given function at the point $(3, 1)$ in the direction of the vector $\mathbf{u} = \langle \mathbf{4}, -\mathbf{3} \rangle$.
   (d) Find an equation of the tangent plane to the graph of the given function at the point $(3, 1, 0)$. Simplify it.

2. The length and width of a rectangle are measured as $30cm$ and $20cm$, respectively, with an error in measurement of at most $0.1cm$ in each. Use differentials to estimate the maximum error in calculated area of the rectangle.

3. Let C be the curve with given vector equation: $\mathbf{r}(\mathbf{t}) = \langle \mathbf{t}, \sqrt{\mathbf{2}} \sin \mathbf{t}, \sqrt{\mathbf{2}} \cos \mathbf{t} \rangle$.
   (a) Find the derivative $\mathbf{r}'(\mathbf{t})$.
   (b) Find parametric equations for the tangent line to C at the point $(\frac{\pi}{4}, 1, 1)$.

4. Consider two planes:
$$x + y - z = 2 \tag{1}$$
   and
$$3x - 4y + 5z = 6. \tag{2}$$
   (a) Find the normal vectors of these planes.
   (b) Find cosine of the angle between the planes.
   (c) Find equations for the line of intersection of these planes.

5. Consider the function
$$f(x, y) = \frac{x^2 + y^2 + 1}{x^2 + y^2 - 1}. \tag{3}$$
   (a) Find and sketch the domain of the function.
   (b) Draw a contour map of the function drawing several level curves.

6. Consider the function
$$z = \ln(x - 3y). \tag{4}$$

(a) Find $z_x(7,2)$ and $z_y(7,2)$.

(b) Find the differential of the function at the point (7,2).

(c) Using linear approximation approximate the value of the function at the point $(6.9, 2.06)$.

(d) Find the rate of change of f at $(7,2)$ in the direction of the vector $\langle 2, -3\rangle$.

(e) Find the maximum rate of change of f at $(7,2)$ and the direction in which it occurs.

7. Show that

$$y\frac{\partial z}{\partial x} - x\frac{\partial z}{\partial y} = 0$$

if $z = f(x^2 + y^2)$.

8. Consider the surface

$$z = x^2 + y^2 - 2x + 6y + 10. \qquad (5)$$

(a) Find an equation of the tangent plane to the given surface at the point $(2, 0, 10)$.

(b) Classify and sketch the graph of given surface.

9. Find z_x and z_y if $y^2 z e^{x+y} - \sin(xyz) = 32$.

10. The radius of a right circular cylinder is decreasing at a rate $1.2 cm/s$ while its height is increasing at a rate of $3cm/s$. At what rate the volume of the cylinder changing when the radius is $80cm$ and the height is $150cm$.

11. The curve C is given by the vector function $\mathbf{r}(t) = \langle 2\cos t, \sin t, t\rangle$. Find a tangent line to C at the point $(0, 1, \pi/2)$.

12. For $f(x, y, z) = x^3 + \sin(xyz)$

(a) find the gradient;

(b) find the directional derivative at the point $(1, \frac{\pi}{2}, 1)$ in the direction of the vector $\langle 2, 1, 2\rangle$;

(c) find the maximum rate of change of f at the point $(1, \frac{\pi}{2}, 1)$.

13. Find the equation of the plane which is tangent to the surface

$$ze^{xyz} = 1$$

at the point $(5, 0, 1)$.

14. Find all critical points of

$$f(x, y) = \frac{1}{3}x^3 + \frac{1}{3}y^3 - 2xy$$

and classify them as local maxima, local minima, or saddle points.

15. Find the point where the line $x = 2 - t, y = 1 + 3t, z = 4t$ intersects the plane $2x - y + z = 4$. At this point $x + y + z =$

a. 9

b. -3

c. 0

d. 3

e. -9

16. The equation of the plane tangent to the surface

$$ze^{\frac{xz}{y}} = 1$$

at $(x, y, z) = (0, \frac{1}{2}, 1)$ is

a. $\frac{1}{2}x + z - 1 = 0$

b. $x = 0$

c. $2x + z - 1 = 0$

d. $2x + y + z = \frac{3}{2}$

e. $2x - y + z = \frac{1}{2}$

17. Use the linear approximation to $f(x, y) = \sqrt{x}e^y$ to estimate (approximate) the value $\sqrt{0.99}e^{0.02}$

a. 1.03

b. 0.975

c. 1.025

d. 1.015

e. 1.01

18. The temperature at a point (x, y, z) is given by

$$T(x, y, z) = e^{-(x+y+z)^3}.$$

The maximum rate of change of T at the point $P(\frac{1}{2}, \frac{1}{4}, \frac{1}{4})$ is

a. $\sqrt{3}$

b. e^{-1}

c. $-3\sqrt{3}e^{-1}$

d. $\sqrt{3}e^{-1}$

e. $3\sqrt{3}e^{-1}$

19. For the function $f(x, y) = xy^2 + x^3 - 2xy$ the point $(x, y) = (\frac{1}{\sqrt{3}}, 1)$ is

a. a local minimum

b. a local maximum

c. a saddle point

d. not a critical point

e. is a critical point but the Second Derivative Test fails.