MATH 251 Extra Practice (EXAM 2)

1. Let
$$f(x, y) = xy - 2x + 5.$$
Find the absolute maximum and minimum values of f on the set D which is the closed triangular region with vertices A(0,0), B(1,1), C(0,1).

2. For
$$\int_0^3 \int_{y^2}^9 f(x, y)\, \mathrm{d}x\mathrm{d}y$$
(a) sketch the region of integration;
(b) change the order of integration.

3. Find the volume of the solid that lies under the paraboloid $z = x^2 + y^2$, above the xy-plane, and inside the cylinder $x^2 + y^2 = 4$.

4. Convert the integral
$$\int_0^1 \int_0^{\sqrt{1-y^2}} \int_{x^2+y^2}^{\sqrt{x^2+y^2}} xyz\, \mathrm{d}z\, \mathrm{d}x\, \mathrm{d}y$$
to an integral in cylindrical coordinates, but don't evaluate it.

5. Let C be the line segment starting at $(0, 1, 1)$ and ending at $(3, 1, 4)$.
a) Find parametric equations for C.
b) Find the mass of a thin wire in the shape of C with the density $\rho(x, y) = x + y$.

6. A particle moves along the curve $C: \quad \vec{r}(t) = \langle t^3, t^2, t\rangle$ from the point $(1, 1, 1)$ to the point $(8, 4, 2)$ due to the force $\vec{F}(x, y, z) = \langle z, y, x\rangle$. Find the work done by the force.

7. Use Green's Theorem to compute the integral along the given positively oriented curve C:
$$\int_C (y^2 - \arctan x)\, \mathrm{d}x + (3x + \sin y)\, \mathrm{d}y,$$
where C is the boundary of the region enclosed by the parabola $y = x^2$ and the line $y = 4$.

8. Convert the integral $\int_0^3 \int_0^{\sqrt{9-y^2}} \int_0^{\sqrt{9-x^2-y^2}} (x^2 + y^2 + z^2)\, \mathrm{d}z\, \mathrm{d}x\, \mathrm{d}y$ to an integral in spherical coordinates, but don't evaluate it.

9. Find the absolute maximum and minimum values of $f(x, y) = x^2y + xy^2 + y^2 - y$ on the set D which is the closed rectangular region in the xy-plane with vertices $(0, 0)$, $(0, 2)$, $(2, 0)$ and $(2, 2)$.

10. Evaluate the integral by reversing the order of integration:

$$\int_0^3 \int_{y^2}^9 y \cos(x^2) \mathrm{dxdy}.$$

11. Find the mass of the lamina that occupies the region bounded by the parabola $x = y^2$ and the line $y = x - 2$ and has the density $\rho(x, y) = 3$.

12. Find the volume of the solid region E in the first octant bounded by the paraboloid $z = x^2 + y^2$, the cone $z = \sqrt{x^2 + y^2}$ and the coordinate planes.

13. Use Green's Theorem to compute the integral

$$\int_C (12 - x^2y - y^3 + \tan x)\, \mathrm{d}x + (xy^2 + x^3 - e^y)\, \mathrm{d}y$$

where C is positively oriented boundary of the region enclosed by the circle $x^2+y^2 = 4$. Sketch the curve C indicating the positive direction.

14. Let $\mathbf{F}(x, y) = \langle 2x + y^2 + 3x^2y, 2xy + x^3 + 3y^2 \rangle$.

    (a) Show that $\mathbf{F}$ is conservative vector field.

    (b) Evaluate $\int_C \mathbf{F} \cdot \mathrm{d}\mathbf{r}$ where C is the arc of the curve $y = x \sin x$ from $(0, 0)$ to $(\pi, 0)$.
    (*Hint:* use the previous part.)

15. Find the mass of the tetrahedron with vertices $(0, 0, 0), (1, 0, 0), (0, 2, 0)$ and $(0, 0, 4)$, if the density is $\rho = x$.

16. Find the volume of the solid that lies under the paraboloid $z = 4 - x^2 - y^2$ and above the xy-plane.

17. Find the line integral of the vector field $\mathbf{F}(x, y, z) = \langle -yz^2, xz^2, z^3 \rangle$ around the circle $\mathbf{r}(t) = \langle 2 \cos t, 2 \sin t, 8 \rangle$.

18. Find the dimensions and volume of the largest rectangular solid box which sits on the xy-plane and has its upper vertices on the paraboloid $z = 4 - 4x^2 - y^2$.

19. Find the mass of the quarter circle $x^2 + y^2 \leq 9$ for $x \geq 0$ and $y \geq 0$ if the density is $\rho(x, y) = \sqrt{x^2 + y^2}$.

20. Find the center of mass of the quarter circle $x^2 + y^2 \leq 9$ for $x \geq 0$ and $y \geq 0$ if the density is $\rho(x, y) = \sqrt{x^2 + y^2}$.

21. Find the volume of the cone $z = 2\sqrt{x^2 + y^2}$ below the paraboloid $z = 8 - x^2 - y^2$.