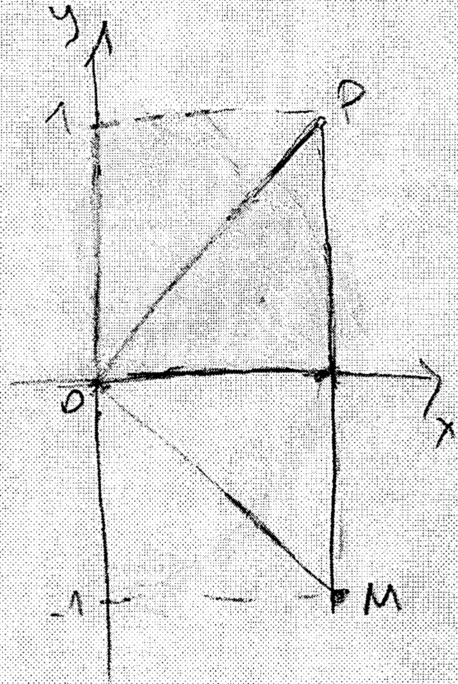
2. Let D be the closed triangular region with vertices $O(0,0)$, $M(1,-1)$, and $P(1,1)$. Find the absolute minimum and absolute maximum values of the function $f(x,y) = 1 - 2x^2 - y^2 - y$ on the region D.



1) Critical points

$$f_x = -4x = 0, \quad f_y = -2y - 1 = 0 \Rightarrow \left(0, -\frac{1}{2}\right) \text{ not in } D$$

2) Corner points

$f(O) = f(0,0) = \boxed{1}$

$f(M) = f(1,-1) = 1-2-1+1 = \boxed{-1}$

$f(P) = f(1,1) = 1-2-1-1 = \boxed{-3}$

Final answer: $\max_D f = \frac{13}{12}$, $\min_D f = -3$

3) OM: $y = -x$, $0 \le x \le 1$

$f|_{OM} = f(x,-x) = 1 - 2x^2 - x^2 + x = 1 - 3x^2 + x = g(x)$

$g'(x) = -6x + 1 = 0 \Rightarrow x = \frac{1}{6}$

$g\left(\frac{1}{6}\right) = f\left(\frac{1}{6}, -\frac{1}{6}\right) = 1 - 3\cdot\frac{1}{6^2} + \frac{1}{6} = 1 - \frac{1}{12} + \frac{1}{6} = \boxed{\frac{13}{12}}$

OP: $y = x$, $0 \le x \le 1$

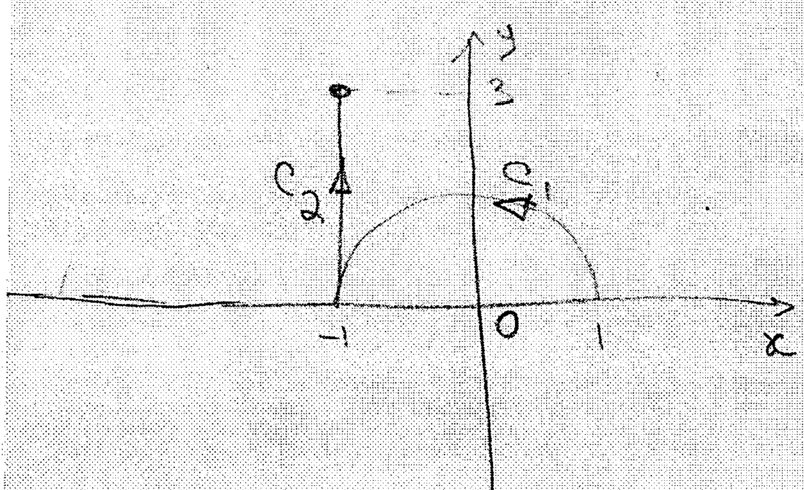
$f|_{OP} = f(x,x) = 1 - 2x^2 - x^2 - x = 1 - 3x^2 - x = h(x)$

$h'(x) = -6x - 1 = 0 \Rightarrow x = -\frac{1}{6}$ → out of the domain of x $(0 \le x \le 1)$

PM: $x = 1$, $-1 \le y \le 1$ $\Rightarrow f|_{PM} = f(1,y) = \underbrace{1-2}_{-1} - y^2 - y = H(y) \Rightarrow H'(y) = -2y - 1 = 0$, $y = -\frac{1}{2}$

$H(-\frac{1}{2}) = f(1, -\frac{1}{2}) = $

30p ddel 1. Evaluate the line integral $\int_C xy^2 dx - y dy$ where C is the upper half of the circle $x^2 + y^2 = 1$ followed by the line segment from the point $(-1, 0)$ to $(-1, 3)$.



$$\int_C xy^2 dx - y dy = \int_{C_1} xy^2 dx - y dy + \int_{C_2} xy^2 dx - y dy$$

C_1: $\quad x = \cos\theta, \quad y = \sin\theta, \quad 0 \le \theta \le \pi$

$\quad dx = -\sin\theta d\theta, \quad dy = \cos\theta d\theta$

$$\int_{C_1} xy^2 dx - y dy = \int_0^\pi \left(\cos\theta \sin^2\theta (-\sin\theta) - \sin\theta \cos\theta\right) d\theta =$$

$$= -\int_0^\pi \cos\theta \sin^3\theta \, d\theta - \int_0^\pi \frac{\sin 2\theta}{2} d\theta =$$

$$= - \frac{\sin^4\theta}{4}\Big|_0^\pi + \frac{\cos 2\theta}{4}\Big|_0^\pi = 0$$

C_2: $\quad x = -1, \quad y = y, \quad 0 \le y \le 3$

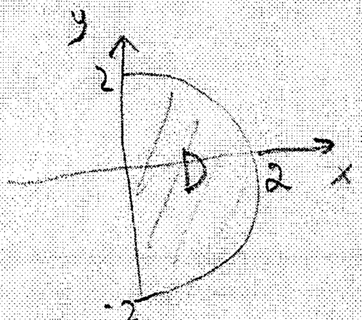
$\quad dx = 0$

$$\int_C xy^2 dx - y dy = -\int_0^3 y \, dy = -\frac{1}{2} y^2 \Big|_0^3 = -\frac{9}{2}$$

$$\int_C xy^2 dx - y dy = \boxed{-\frac{9}{2}}$$

3. Find the absolute minimum and absolute maximum values of $f(x,y) = e^{x^2+y^2-xy}$ on the region

$$D = \{(x,y) | x^2+y^2 \le 4, x \ge 0\}.$$



$$\begin{cases} f_x = (2x-y)e^{x^2+y^2-xy} = 0 \\ f_y = (2y-x)e^{x^2+y^2-xy} = 0 \end{cases} \Rightarrow \begin{cases} 2x-y=0 \\ 2y-x=0 \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} y=2x \\ 4x-x=0 \end{cases} \Rightarrow \underbrace{\boxed{(0,0)}}_{\text{in } D} \quad f(0,0) = e^0 = \boxed{1}$$

2) Corner points: $f(0,2) = \boxed{e^4}$ $\quad f(0,-2) = e^4$

3) Boundary

a) Left segment

$y=0$, $-2 \le x \le 2$

$f(x,0) = e^{x^2} = g(x)$

$g'(x) = 2xe^{x^2} = 0$

$x=0 \Rightarrow f(0,0) = 1$

b) Right semicircle

$x = 2\cos\theta$, $y = 2\sin\theta$ $\quad -\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

$f(2\cos\theta, 2\sin\theta) = e^{4-4\sin\theta\cos\theta} =$

$= e^{4-2\sin 2\theta} = h(\theta)$

$h'(\theta) = -4\cos 2\theta \, e^{4-2\sin 2\theta} = 0$

$-4\cos 2\theta = 0$

$\cos 2\theta = 0 \Rightarrow 2\theta = \frac{\pi}{2} + \pi n \Rightarrow$

$\theta = \frac{\pi}{4} + \frac{\pi n}{2}$

Since $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$,

we get $\theta = -\frac{\pi}{4}$ and $\frac{\pi}{4}$

$h\left(\frac{\pi}{4}\right) = \boxed{e^2}$ $\rightarrow$ corresp to the point $(\sqrt{2}, \sqrt{2})$

$h\left(-\frac{\pi}{4}\right) = \boxed{e^6}$

corresponds to the point $(\sqrt{2}, -\sqrt{2})$

Final answer

$$\boxed{\max_D f = e^6 \qquad \min_D f = 1}$$