F19_LN_13_4

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13.4: Motion in Space: Velocity and Acceleration (very shortly as most of this material was in principle discussed in the previous two sections)

Suppose a particle moves through space so that its position vector at time t is $\mathbf{r}(t)$. Assume that $\mathbf{r}(t)$ is twice differentiable, i.e. each component has all derivatives up to order 2 at every point.

- The *velocity* of the particle at t is $\mathbf{v}(t) := \mathbf{r}'(t)$.
- The *speed* of the particle at t is the magnitude of the velocity, $|\mathbf{v}(t)| = |\mathbf{r}'(t)|$.
- The *acceleration* pf the particle at t is $\mathbf{a}(t) = \mathbf{v}'(t) = \mathbf{r}''(t)$.

EXAMPLE 1. *Find the velocity, speed, and acceleration of a particle with the given position function*

$$\mathbf{r}(t) = -\sqrt{2}t\,\mathbf{i} + e^t\,\mathbf{j} + e^{-t}\,\mathbf{k}$$

$$\vec{v}(t) = r'(t) = -\sqrt{2}\,\hat{i} + e^t\,\hat{j} - e^{-t}\,\hat{k}$$

$$\text{Speed} = |\vec{v}(t)| = \sqrt{\underbrace{\underbrace{2}_{2e^t e^{-t}} + \underbrace{e^{2t}}_{(e^t)^2} + \underbrace{e^{-2t}}_{(e^{-t})^2}}_{(e^t+e^{-t})^2}} = e^t + e^{-t}$$

$$\vec{a}(t) = \vec{v}\,'(t) = 0\,\hat{i} + e^t\,\hat{j} + e^{-t}\,\hat{k} = e^t\,\hat{j} + e^{-t}\,\hat{k}$$

EXAMPLE 2. *Find the position vector of a particle that has the given acceleration and the specified initial velocity and position.*

$$\mathbf{a}(t) = (t^2 - t)\mathbf{i} + \cos 3t\mathbf{j} + e^{-2t}\mathbf{k},\ \ \mathbf{v}(0) = 2\mathbf{i} - 3\mathbf{j},\ \ \mathbf{r}(0) = 3\mathbf{i} + 2\mathbf{j} - \mathbf{k}$$

$$\vec{v}(t) = \int_0^t \vec{a}(\tau)\,d\tau + \vec{v}(0) = \int_0^t (\tau^2 - \tau)\,d\tau\,\hat{i} +$$

$$+\int_0^t \cos 3\tau \, d\tau \, \hat{j} + \left(\int_0^t e^{-2\tau} \, d\tau\right)\hat{k} + 2\hat{i} - 3\hat{j} = \left(\frac{t^3}{3} - \frac{t^2}{2} + 2\right)\hat{i} +$$

$$+\left(\frac{1}{3}\sin 3t - 3\right)\hat{j} + \underbrace{-\frac{1}{2} e^{-2\tau}\Big|_0^t \hat{k}}_{\left(-\frac{1}{2} e^{-2t} + \frac{1}{2}\right)\hat{k}}$$

$$\vec{r}(t) = \int_0^t v(\tau)\, d\tau + \vec{r}(0) = \left(\frac{t^4}{12} - \frac{t^3}{6} + 2t + 3\right)\hat{i} +$$

$$+\left(-\frac{1}{9}\cos 3\tau \Big|_0^t - 3t + 2\right)\hat{j} + \left(\frac{1}{4} e^{-2\tau}\Big|_0^t + \frac{1}{2}t - 1\right)\hat{k} =$$

$$= \left(\frac{t^4}{12} - \frac{t^3}{6} + 2t + 3\right)\hat{i} + \underbrace{\left(-\frac{1}{9}\cos 3t + \frac{1}{9} - 3t + 2\right)\hat{j}}_{\left(-\frac{1}{9}\cos 3t - 3t + \frac{19}{9}\right)\hat{j}} + \underbrace{\left(\frac{1}{4}e^{-2t} - \frac{1}{4} + \frac{1}{2}t - 1\right)\hat{k}}_{\left(\frac{1}{4}e^{-2t} + \frac{1}{2}t - \frac{5}{4}\right)\hat{k}}$$