



F19_LN_1...

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13.4: Motion in Space: Velocity and Acceleration (very shortly as most of this material was in principle discussed in the previous two sections)

Suppose a particle moves through space so that its position vector at time t is $\mathbf{r}(t)$. Assume that $\mathbf{r}(t)$ is twice differentiable, i.e. each component has all derivatives up to order 2 at every point.

- The *velocity* of the particle at t is $\mathbf{v}(t) := \mathbf{r}'(t)$.
- The *speed* of the particle at t is the magnitude of the velocity, $|\mathbf{v}(t)| = |\mathbf{r}'(t)|$.
- The *acceleration* of the particle at t is $\mathbf{a}(t) = \mathbf{v}'(t) = \mathbf{r}''(t)$.

EXAMPLE 1. Find the velocity, speed, and acceleration of a particle with the given position function

$$\mathbf{r}(t) = -\sqrt{2}t\mathbf{i} + e^t\mathbf{j} + e^{-t}\mathbf{k}$$

$$\mathbf{v}(t) = \mathbf{r}'(t) = -\sqrt{2}\hat{\mathbf{i}} + e^t\hat{\mathbf{j}} - e^{-t}\hat{\mathbf{k}}$$

$$\text{Speed} = |\mathbf{r}'(t)| = \sqrt{2 + e^{2t} + e^{-2t}} = e^t + e^{-t}$$

$\frac{2 \cdot e^t \cdot e^{-t}}{2ab} \quad \frac{(e^t)^2}{b^2} \quad \frac{(e^{-t})^2}{(e^{-t})^2} \rightarrow a^2 \quad (e+b)^2 = e^2 + 2eb + b^2$

$$\mathbf{a}(t) = \mathbf{v}'(t) = 0\hat{\mathbf{i}} + e^t\hat{\mathbf{j}} + e^{-t}\hat{\mathbf{k}} = e^t\hat{\mathbf{j}} + e^{-t}\hat{\mathbf{k}}$$

EXAMPLE 2. Find the position vector of a particle that has the given acceleration and the specified initial velocity and position.

$$\mathbf{a}(t) = (t^2 - t)\mathbf{i} + \cos 3t\mathbf{j} + e^{-2t}\mathbf{k}, \quad \mathbf{v}(0) = 2\mathbf{i} - 3\mathbf{j}, \quad \mathbf{r}(0) = 3\mathbf{i} + 2\mathbf{j} - \mathbf{k}$$

$$\vec{v}(t) = \int_0^t \vec{a}(\tau) d\tau + \vec{v}(0) = \left(\int_0^t (\tau^2 - \tau) d\tau + 2 \right) \hat{\mathbf{i}} + \dots$$

$$\begin{aligned} \vec{v}(t) &= \int \vec{a}(\tau) d\tau + \vec{v}(0) = \left(\int (\tau^2 - \tau) d\tau + 2 \right) \hat{i} + \\ &+ \left(\int_0^t \cos 3\tau d\tau - 3 \right) \hat{j} + \left(\int_0^t e^{-2\tau} d\tau \right) \hat{k} = \left(\frac{t^3}{3} - \frac{t^2}{2} + 2 \right) \hat{i} + \\ &+ \left(\frac{1}{3} \sin 3\tau \Big|_0^t - 3 \right) \hat{j} + \left(-\frac{1}{2} e^{-2\tau} \Big|_{\tau=0}^t \right) \hat{k} = \left(\frac{t^3}{3} - \frac{t^2}{2} + 2 \right) \hat{i} + \\ &+ \left(\frac{1}{3} \sin 3t - 3 \right) \hat{j} + \left(-\frac{1}{2} e^{-2t} + \frac{1}{2} \right) \hat{k} \end{aligned}$$

$$\begin{aligned} \vec{r}(t) &= \int \vec{v}(\tau) d\tau + \vec{r}(0) = \left(\frac{t^4}{12} - \frac{t^3}{6} + 2t + 3 \right) \hat{i} + \\ &+ \left(-\frac{1}{9} \cos 3\tau \Big|_0^t - 3t + 2 \right) \hat{j} + \left(\frac{1}{4} e^{-2\tau} \Big|_0^t + \frac{1}{2} t - 1 \right) \hat{k} = \\ &= \left(\frac{t^4}{12} - \frac{t^3}{6} + 2t + 3 \right) \hat{i} + \left(-\frac{1}{9} \cos 3t + \frac{1}{9} - 3t + 2 \right) \hat{j} + \left(\frac{1}{4} e^{-2t} - \frac{1}{4} + \frac{1}{2} t - 1 \right) \hat{k} \end{aligned}$$