Version A (blue)

1. [50pts] Find z_y if $xy^2z^3 + x^3y^2z - x - y - z = 33$

2. [50pts] Find the directional derivative of the function $f(x, y, z) = 12 - y^2z + x^3$ at the point $(2, 1, 6)$ in the direction of the vector $\mathbf{u} = \langle -4, 4, 2\rangle$.

1. Let $F = xy^2z^3 + x^3y^2z - x - y - z$

Then our function z is given implicitely by $F(x,y,z) = 3$

We use the formula

$$z_y = -\frac{F_y}{F_z} \quad (1)$$

Calculate F_y and F_z:

$$F_y = 2xyz^3 + 2x^3yz - 1$$

$$F_z = 3xy^2z^2 + x^3y^2 - 1$$

Plug into (1): $\boxed{z_x = -\frac{2xyz^3 + 2x^3yz - 1}{3xy^2z^2 + x^3y^2 - 1}}$

2. i) Normalize u: $\hat{u} = \frac{\langle -4, 4, 2\rangle}{\sqrt{36}} = \langle -\frac{2}{3}, \frac{2}{3}, \frac{1}{3}\rangle$

ii) Find the gradient of f at $(2,1,6)$

$$\nabla f(x,y,z) = \langle 3x^2, -2yz, -y^2\rangle$$

$$\nabla f(2,1,6) = \langle 12, -12, -1\rangle$$

iii) Use the formula $D_{\hat{u}} f = \nabla f \cdot \hat{u}$

$D_{\hat{u}} f(2,1,6) = \langle 12, -12, -1\rangle \cdot \langle -\frac{2}{3}, \frac{2}{3}, \frac{1}{3}\rangle = 12 \cdot (-\frac{2}{3}) + (-12)\frac{2}{3} + (-1)\cdot\frac{1}{3} = -8 - 8 - \frac{1}{3} = \boxed{-16\frac{1}{3}} = \boxed{-\frac{49}{3}}$