Workshop on "Geometry of vector distributions, differential equations, and variational problems"

SISSA, Trieste, Italy, 13–15 December 2006

Abstracts of the talks

Dmitri Alekseevsky (University of Hull, UK) *Homogeneous bi-Lagrangian manifolds of semisimple group and generalized Gauss decompositions*

The talk is bases on a joint work with C.Medori (Parma).

A bi-Lagrangian structure on a (real or complex) symplectic manifold (M, ω) is a decomposition $TM = T^+ + T^-$ of the tangent bundle TM into a direct sum of integrable Lagrangian ($\omega|_{T^\pm} = 0$) subbundles $T^\pm$. In the real case, a manifold M with a bi-Lagrangian structure $(\omega, T^\pm)$ can be identified with a para-Kähler manifold (M, g, J) where $J \in \Gamma(\mathrm{End}TM)$ is the involutive endomorphism with ± 1-eigenspace distributions $T^\pm$ and $g = \omega \circ J$ is the pseudo-Riemannian metric such that the endomorphism J is g-skew-symmetric and parallel with respect to Levi-Civita connection of g.

The problem of classification of bi-Lagrangian manifolds $(M, \omega, T^\pm)$ which admit a semisimple transitive Lie group G of automorphisms reduces to a description of generalized Gauss decompositions

$$\mathfrak{g} = \mathfrak{k} + \mathfrak{m}^+ + \mathfrak{m}^-$$

of the Lie algebra $\mathfrak{g}$ of G where $\mathfrak{p}^\pm := \mathfrak{k} + \mathfrak{m}^\pm$ are opposite parabolic subalgebras with the reductive part $\mathfrak{k}$ which is the stability subalgebra of $\mathfrak{g}$. We give a description of such decomposition of a complex semisimple Lie algebra $\mathfrak{g}$ in terms of crossed Dynkin diagrams and a real semisimple Lie algebra $\mathfrak{g}$ in terms of crossed Satake diagrams.