Canonical frames for vector distributions of rank two and three

Igor Zelenko

SISSA, Trieste, Italy

This is the joint work with Boris Doubrov. First we will describe a new rather effective procedure of symplectification for the problem of local equivalence of non-holonomic vector distributions. The starting point of this procedure is to lift a distribution D to a special submanifold W_D of the cotangent bundle, foliated by the characteristic curves (the abnormal extremals of the distribution D). In particular, if D is a rank 2 distribution then the submanifolds W_D is nothing but the annihilator of the square of D, while if D is a distribution of odd rank it is the annihilator of D itself. The dynamics of the lifting (to W_D) of the distribution D along the characteristic curves (of W_D) is described by certain curves of flags of isotropic and coisotropic subspaces in a linear symplectic space. So, the problem of equivalence of distributions can be essentially reduced to the differential geometry of such curves: the invariants of these curves are automatically invariants of the distribution D and the canonical frame bundles, associated with such curves, can be in many cases effectively used for the construction of the canonical frames of the distributions D itself on certain fiber bundles over W_D. In this way we succeeded to construct the canonical frames for germs of rank 2 distributions in $\mathbb{R}^n$ with $n > 5$ and of rank 3 distributions in $\mathbb{R}^7$ from certain generic classes. The first case generalizes the classical work of E. Cartan (1910) on rank 2 distributions in $\mathbb{R}^5$. The second case is also new: the only rank 3 distributions, treated before, were rank 3 distributions in $\mathbb{R}^5$ (Cartan, 1910) and in $\mathbb{R}^6$ (N. Tanaka school and independently R. Bryant in 70th). In all these cases the most symmetric models will be given as well.