On the Geometry of Hypersurfaces of Conullity Two in Euclidean Space

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We give a geometric description of the class $\mathfrak{K}_\mathrm{o}$ of hypersurfaces of conullity two with involutive geometric two-dimensional distributions proving that the integral surfaces of these distributions are surfaces with flat normal connection, which are not developable and conversely, that any two-dimensional surface with flat normal connection, which is not developable, generates a hypersurface of conullity two from the class $\mathfrak{K}_\mathrm{o}$. In this way the hypersurfaces of conullity two from the class $\mathfrak{K}_\mathrm{o}$ are in one-to-one correspondence with the two-dimensional surfaces with flat normal connection, which are not developable.

We characterize the hypersurfaces of conullity two also as envelopes of two-parameter families of hyperplanes, proving that a hypersurface in Euclidean space is locally a hypersurface of conullity two if and only if it is the envelope of a two-parameter family of hyperplanes. This geometric characterization allows us to obtain a parametrization of each hypersurface of conullity two by a pair of a unit vector function $l(u, v)$ and a scalar function $r(u, v)$. We obtain a characterization in terms of a system of partial differential equations for the geometric functions $l(u, v)$ and $r(u, v)$ of two main classes of hypersurfaces of conullity two: minimal hypersurfaces of conullity two and hypersurfaces of conullity two of umbilical type.