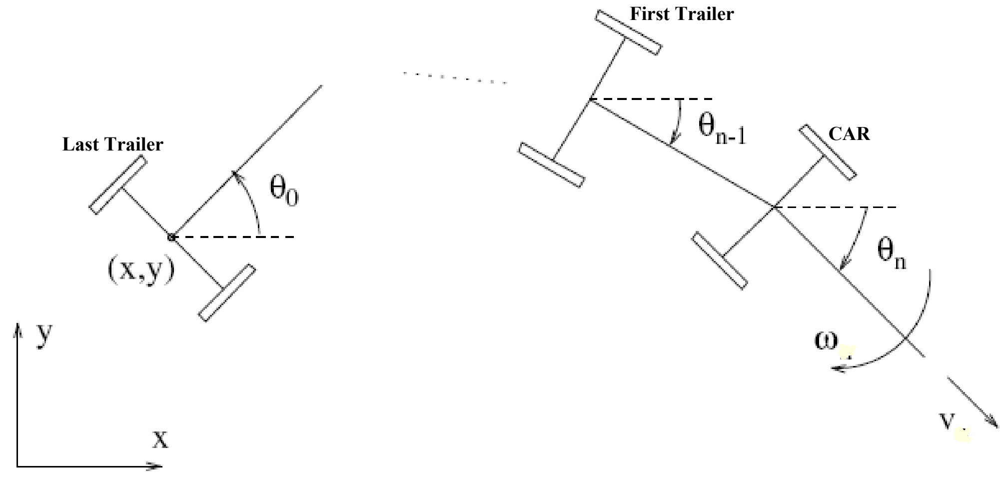
Homework Assignment 1 in Geometric Control Theory, MATH666

due to Sept 23, 2011



Problem Consider the system of a car with n direct-hooked passive trailers (see the Figure). A car and trailers are represented by two driving wheels connected by an axle. Each trailer is hooked up at the middle point of the wheel axle of the previous body by a rigid bar of length 1. Assume that (x, y) are coordinates of the middle point of the axis of the last trailer, θ_n is the angle of the pair of the wheel of the car with respect to the x-axis, and θ_i, $0 \leq i \leq n-1$, are the angles of the pair of wheels of the $(n-i)$th trailer with respect to the x-axis. The state of the system is parametrized by $q = (x, y, \theta_0, \ldots, \theta_n)$, i.e. it is $R^2 \times \underbrace{S^1 \times \ldots \times S^1}_{n+1 \text{ times}} = R^2 \times \mathbb{T}^{n+1}$, where $\mathbb{T}^{n+1}$ is the $(n+1)$-dimensional torus. The wheels of each body are constrained to roll without slipping, i.e. the velocity of each body is in the direction parallel to the direction of the wheels. We control the linear velocity of the car (by the control v) and the angular velocity of the car (by the control ω). Assume that $f_n = 1$ and

$$f_i = \cos(\theta_{i+1} - \theta_i)\cos(\theta_{i+2} - \theta_{i+1})\ldots\cos(\theta_n - \theta_{n-1}) = \prod_{j=i+1}^{n} \cos(\theta_j - \theta_{j-1}), \quad 0 \leq i \leq n-1.$$

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a. Show that the control system corresponding to this situation has the form:

$$\dot{q} = \omega X_1(q) + v X_2(q),$$

where

$$\begin{aligned} X_1 &= \frac{\partial}{\partial \theta_n}, \\ X_2 &= \cos\theta_0 f_0 \frac{\partial}{\partial x} + \sin\theta_0 f_0 \frac{\partial}{\partial y} + \sin(\theta_1 - \theta_0) f_1 \frac{\partial}{\partial \theta_0} + \ldots + \\ &\quad \sin(\theta_{i+1} - \theta_i) f_{i+1} \frac{\partial}{\partial \theta_i} + \ldots \sin(\theta_n - \theta_{n-1}) f_n \frac{\partial}{\partial \theta_{n-1}} \end{aligned}$$

b. Assume that we have a car with one trailer, i.e. $n = 1$. Prove that the control system is controllable on $\mathbb{R}^2 \times \mathbb{T}^2$ by piecewise constant control functions.