Homework Assignment 3 in Geometric Control Theory, MATH666

due Nov. 25, 2013

1. Consider the control system

$$\begin{cases} \dot{x}_1 = 8x_1 + 3x_2 + 2u \\ \dot{x}_2 = -10x_1 - 3x_2 - u \end{cases}, -4 \leq u \leq 2.$$

Find the attainable set $\mathcal{A}_{(0,0)}$ from the point $(0,0)$ and draw the time optimal synthesis with the target $(0,0)$ in the plane (x_1, x_2).

2. (a) (Optimal U-turn of the Dubins car) Consider the time-minimal problem for the control system corresponding to the Dubins car:

$$\begin{cases} \dot{x}_1 = \cos\theta \\ \dot{x}_2 = \sin\theta \\ \dot{\theta} = u \end{cases}, \quad |u| \leq 1. \tag{1}$$

Find the time-optimal control and trajectory for the boundary conditions:

$$\big(x_1(0), x_2(0), \theta(0)\big) = (0,0,0), \quad \big(x_1(t_1), x_2(t_1), \theta(t_1)\big) = (0,0,\pi).$$

You can use the description of extremal controls and trajectories for Dubins car time-minimal problem given in class (see also section 13.5 of the textbook).

(b) (Dubins car with the steering wheel turning counterclockwise only) Consider the time-minimal problem with the fixed initial and terminal points for the same control system as in (1) except that the control satisfies $0 \leq u \leq 1$, i.e. one can turn the steering wheel counterclockwise only. Describe all extremal controls and trajectories (for all possible fixed initial and terminal points).

(c) Find the optimal control and trajectory in the problem of item (b) for the following boundary conditions:

$$\big(x_1(0), x_2(0), \theta(0)\big) = (0,0,0), \quad \big(x_1(t_1), x_2(t_1), \theta(t_1)\big) = \left(0, 0, \frac{5\pi}{4}\right)$$