

We first need to find \mathbf{F} so that

$$\int \int_S \mathbf{F} \cdot \hat{\mathbf{n}} \, dS = \int \int_S (ax + by + z^2) \, dS,$$

so

$$\mathbf{F} \cdot \hat{\mathbf{n}} = ax + by + z^2.$$

But for S we have:

$$\hat{\mathbf{n}} = \left\langle \frac{x}{\sqrt{x^2 + y^2 + z^2}}, \frac{y}{\sqrt{x^2 + y^2 + z^2}}, \frac{z}{\sqrt{x^2 + y^2 + z^2}} \right\rangle = \langle x, y, z \rangle$$

because $x^2 + y^2 + z^2 = 1$. Thus \mathbf{F} can be taken as $\mathbf{F} = \langle a, b, z \rangle$.